

Motion at Constant Acceleration Along a Straight Line

Part I: Motion at Constant Velocity

Motion at constant velocity means motion in which the speed and the direction of the motion are constant. If the direction is constant, it means that the motion is necessarily along a straight line, i.e. there are no turns. The straight line can be horizontal, vertical, or anything in between. The direction of the line along which motion occurs is exactly the direction of the velocity.

The equation for position, x , as a function of time, t , for the case of constant velocity, v , is

$$x(t) = x_0 + v \cdot t$$

where x_0 is the position at time $t=0$. We assume here that x_0 and v are along the same straight line.

A very simple example of constant velocity motion can be observed here:

<https://phet.colorado.edu/sims/cheerpi/moving-man/latest/moving-man.html?simulation=moving-man>

In this case, the motion is horizontal.

Exercise 1:

Select the “Charts” tab at the top of the simulation window. Put a check mark in the “show vector” boxes for both velocity and acceleration. Try the following combinations of initial position and velocity. Leave the acceleration as 0 for all cases. (Note, values do not have to be whole numbers.)

- A. Positive initial position, negative velocity
- B. Negative initial position, positive velocity

For each case,

- i. Record your choice of initial position and velocity.
- ii. Take a screen shot after the figure reaches the brick wall.
- iii. Write down the equation which corresponds to this motion (the equation should be valid up to, but not including the moment the figure hits the wall.)
- iv. Check that your equation is correct by using the “expression evaluator” feature. Go to the “special features” menu option, at the top of the window. Click on “expression evaluator”. Click “Clear”. Enter your equation, and “play” the resulting motion. Make sure to be careful with +- signs and write the multiplication sign * explicitly. If the “expression evaluator” window disappears, click the Introduction tab and then click the Charts tab.
- v. Use your equation to find the time at which the figure hits the wall to 3 significant figures.

- vi. Qualitatively describe the velocity and acceleration graphs around the moment the figure hits the wall. Is the velocity constant? Is the acceleration 0, positive, or negative? Is the direction of the acceleration same as the direction of the velocity or opposite?

If you are not sure how to write the equation, review the relevant theory in your textbook:

<https://openstax.org/books/college-physics/pages/2-5-motion-equations-for-constant-acceleration-in-one-dimension>

Exercise 2:

In the previous simulation, the motion was horizontal. But we can have constant velocity motion in any direction. Run the following simulation:

<https://phet.colorado.edu/sims/cheerpi/ladybug-motion-2d/latest/ladybug-motion-2d.html?simulation=ladybug-motion-2d>

On the right side of the window, select “show both” under **Vectors**, select “velocity” in the bottom **Remote Control** section. It is easiest to control the ladybug using the **Remote Control**. Press and hold the left mouse button on the dot in the middle of the **Remote Control** square. You can rotate the velocity vector and adjust its length. The length represents the speed. Choose a speed that’s not too large, so that you have time to observe the motion. Choose a direction that’s not strictly horizontal or vertical. After you’ve set your initial velocity, click “Linear” in the “**Choose Motion**” section. (Note that you must reselect “Linear” after any adjustment with the remote control in order to initiate the motion with the adjusted velocity.)

You should see the ladybug move across the screen along straight lines, except for the moments when it bounces off the walls. The green vector in front of the ladybug represents the velocity. Its direction is along the direction of motion. When the ladybug bounces off the walls, you should briefly see a large pink vector, which represents the acceleration. You should not see a pink vector at other times since the motion is at constant velocity i.e. 0 acceleration. The velocity in this situation is described as “piecewise constant”. Between the bounces, the motion is along straight lines, but overall the motion is in “two dimensions”. We will study motion in two dimensions in more generality in subsequent labs.

- i. Take a screen shot at a moment when the ladybug is not very close to a wall
- ii. Look at the trace of the motion and state how many constant velocity segments are visible in the screen shot.

Part II: Motion at Constant Acceleration

We now consider motion at constant acceleration along a straight line. Motion at constant acceleration means that the rate at which velocity changes is constant and the direction of that change is also constant. Make sure not to confuse motion at constant acceleration and motion at constant velocity.

In order for the motion to be along a straight line, there is a further restriction that the velocity and acceleration must be colinear. If the velocity and acceleration vectors are at an angle to each other, the motion is two dimensional. We will discuss this situation in a later lab.

Motion at constant acceleration describes one of the most common processes in our daily life: objects falling. To a good approximation (\pm half a percent), the acceleration caused by the force of Earth's gravity does not depend on altitude, for altitudes inside the stratosphere. Therefore, a falling object experiences constant gravitational acceleration. Moreover, an object which has been thrown also experiences constant gravitational acceleration from the moment it is released until it lands. That same constant gravitational acceleration is experienced by the object, during upward motion and during downward motion. Of course, the *velocity is not constant. The velocity is changing.*

The gravitational acceleration is not only constant, but is also *exactly* the same for any object. When you jump, while you are in the air, you experience exactly the same gravitational acceleration as experienced by a ball that you have thrown, while it is in the air. The gravitational acceleration near the Earth's surface is 9.8m/s^2 (to two significant figures) and is directed "down" i.e. towards the Earth's center.

Before we proceed with a quantitative exploration of free fall (which is the term for motion with only gravitational acceleration), there is a caveat to address. Gravitational acceleration is constant and identical for all objects. However, falling and thrown objects, that we observe in everyday life, are not exactly in free fall, i.e. they are not experiencing gravitational acceleration only. They also experience air resistance. The acceleration due to air resistance is not constant and is highly dependent on the specific characteristics of an object, i.e. its size, shape, and mass.

For most objects, in situations that do not involve high speeds, the effect of air resistance is small and can be ignored to a good approximation. These are the cases that we will concern ourselves with in this lab.

The equation for the position, y , as a function of time, t , for motion at constant acceleration, a , is

$$y(t) = y_0 + v_0t + \frac{1}{2}at^2$$

where y_0 and v_0 are the values of the position and the velocity, respectively, at $t=0$. We assume here that y_0 , v_0 , and a are along the same straight line.

We can also relate the velocity and the position without reference to time as follows

$$v_2^2 - v_1^2 = 2a(y_2 - y_1)$$

Where v_2 is the velocity at position y_2 and v_1 is the velocity at position y_1 .

Exercise 3:

Run the following simulation:

https://phet.colorado.edu/sims/html/projectile-motion/latest/projectile-motion_en.html

Select the “Intro” option on the bottom of the window. The “air resistance” option should be unchecked. Select the option to display both the velocity and acceleration vectors. (It does not matter whether you choose “total” or “components” since we will only consider purely vertical motion.)

The height of the cannon can be raised by clicking on it and dragging the mouse upwards while keeping it pressed. Raise the cannon to a height of 8-15m. Set the initial speed to a value 1-10m/s. Point the cannon directly down, i.e. at -90 degrees. Select the “slow” option. Launch. Check that the trajectory has at least 10 measurement points. If you have fewer than 10 points, raise the height of the cannon and/or lower the initial speed. The eraser button clears the trajectory. Note that you can zoom in to make it easier to take readings.

- i. Make sure to record your choice of initial height and initial speed.
- ii. State whether the velocity and acceleration point in the same direction.
- iii. Take a screen shot of the trajectory.
- iv. Use the time & height “measurement” tool to record at least 10 corresponding time & height “measurements”, by placing the circle on the marked points along the trajectory. In excel, make a column of times and a column of corresponding heights.
- v. Make a scatter plot with time on the x axis and height on the y axis.
- vi. Add a trendline, using the polynomial fit of order 2. Make sure to display the equation.
- vii. Based on your knowledge motion at constant acceleration, write down the equation for position as a function of time, with your chosen initial position, your chosen initial velocity, and Earth’s gravitational acceleration. (Be careful with \pm signs.)

If you are not sure how to write the equation, review the relevant theory in your textbook:

<https://openstax.org/books/college-physics/pages/2-7-falling-objects>

- viii. Add a column in excel, in which you generate heights for the times from your simulation that you recorded earlier.
- ix. Describe how well do the values generated by your equation match the values generated by the simulation? How closely does the trendline from the graph of the simulation data match your equation? You do not have to quantify the exactness of the match, since the simulation should match your calculation very closely. If you do not have a close match, check your work.
- x. Label the axis of your plot with units and denote time vs height. Give your chart a title.
- xi. Discuss whether the results of simulation would change if a different object was chosen (assuming air resistance remains turned off).

Exercise 4:

Set the cannon to a height of 1-7m. Set the initial speed to a value $>10\text{m/s}$. Point the cannon directly up, i.e. at $+90$ degrees. Select the “slow” option. Launch. Make sure to record your choice of initial height and initial speed.

- i. State whether the direction of the velocity changes or stays the same throughout the motion. What about the direction of the acceleration? Describe how the direction of the velocity and the acceleration compare throughout the motion (are they same, different, sometimes same, sometimes different?)
- ii. Take a screen shot of the trajectory.
- iii. Find the highest point and record its time and height.
- iv. Find the lowest point and record its time and height.
- v. Based on your knowledge motion at constant acceleration, write down the equation for position as a function of time, with your chosen initial position, your chosen initial velocity, and Earth’s gravitational acceleration. (Be careful with \pm signs.)

If you are not sure how to write the equation, review the relevant theory in your textbook:

<https://openstax.org/books/college-physics/pages/2-7-falling-objects>

- vi. Make a column of time values with 0.1 second intervals, from $t=0$ up to the highest time value in your plot. (You can generate the time values automatically by entering 0 into slot A1 and the formula “ $=A1+0.1$ ” into slot A2 and then dragging your cursor down until you have the desired values. Use the equation that you wrote down to generate height values in the second column.
- vii. Make a scatter plot with time on the x axis and height on the y axis.

- viii. Add a trendline, using the polynomial fit of order 2. Make sure to display the equation and check that it matches the equation you wrote down in step v.
- ix. Does the time and height value of the highest point on the graph match the simulation?
- x. Does the time and height value of the lowest point, match the simulation?
- xi. Choose 5 points in the simulation on the way up and 5 points in the simulation on the way down and record their heights in an excel column, at the corresponding time.
- xii. Describe how well do the values generated by your equation match the values generated by the simulation?
- xiii. Label the axis of your plot with units and denote time vs height. Give your chart a title.
- xiv. Use the equation that directly relates velocity and position without reference to time. Use your $t=0$ values for y_1 and v_1 . Use the highest point for y_2 . Solve for v_2 . Explain whether your answer makes sense as the value of the velocity at the highest point?